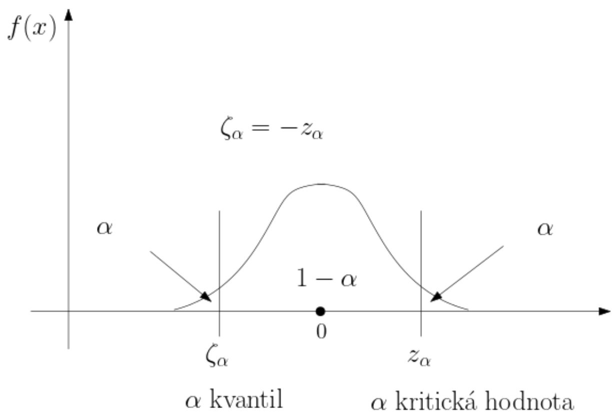
Kvantil a kritická hodnota (standardní normální rozdělení)



$\int_{-\infty}^{\zeta_\alpha} f(x)\,dx = \alpha$ $\qquad$ $\int_{z_\alpha}^{\infty} f(x)\,dx = \alpha$

Bodové odhady

Příklad

Máme generátor spojitých náhodných čísel s předpokládaným normálním rozdělením s rozptylem $s^2 = 1$, ale stření hodnotu μ neznáme a chceme ji odhadovat. Pro odhad potřebujeme statistiku T a její rozdělení $f(T)$. Je to

$$T = \frac{1}{N}\sum x_i \quad a \quad f(T) = N_T\left(\mu, \frac{\sigma^2}{N}\right)$$

ale dejme tomu, že jí neznáme. Jak se k ní dostaneme?

Výběr $\quad x = [x_1, x_2, \cdots, x_N]$

Odhad

– **metoda momentů** (moment souboru = moment výběru)

$$\mu = \frac{1}{N}\sum x_i$$

– **metoda maximální věrohodnosti** (maximum likelihood)

Likelihood = hustota výběru jako funkce parametrů $\left[f(x) = \prod_{i=1}^{N} f(x_i)\right]$

$$L_N(\mu) = \prod_{i=1}^{N} f(x_i)$$

a v maximu L leží bodový odhad μ.

Model

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-0.5\,(x-\mu)^2\right\} \propto \exp\left\{-0.5\left[x^2 - 2\mu x + \mu^2\right]\right\}$$

Součin dvou modelů

$$\exp\left\{-0.5\left[x_1^2-2\mu x_1+\mu^2\right]\right\}\exp\left\{-0.5\left[x_2^2-2\mu x_2+\mu^2\right]\right\}=$$

$$=\exp\left\{-0.5\left[\underbrace{x_1^2+x_2^2}_{R_2}-2\mu\left(\underbrace{x_1+x_2}_{S_2}\right)+\underbrace{2}_{\kappa_2}\mu^2\right]\right\}=$$

$$=\exp\left\{-0.5\left(R_2-2\mu S_2+\kappa_2\mu^2\right)\right\}$$

$\cdots$ pro t modelů bude

$$L_t(\mu)\propto\exp\left\{-0.5\left(R_t-2\mu S_t+\kappa_t\mu^2\right)\right\}$$

kde

$$R_t=R_{t-1}+x_t^2$$
$$S_t=S_{t-1}+x_t$$
$$\kappa_t=\kappa_{t-1}+1$$

Maximum L (derivace = 0) $\quad\cdots\quad \frac{d}{dx}\exp\{g(\mu)\}=\exp\{g(\mu)\}\times\frac{d}{d\mu}g(\mu)$

$$\frac{d}{d\mu}L=L\times\frac{d}{d\mu}\text{exponent}=L\times[-0.5(-2S_t+2\kappa_t\mu)]=0$$

$$\hat{\mu}=\frac{S_t}{\kappa_t}=\frac{\sum x_i}{t}\quad\text{pro}\quad t=N$$

Exponenciální rozdělení

Model

$$f(x)=a\exp\{-ax\}$$

Součin dvou

$$a^2\exp\{-a(x_1+x_2)\}$$

Likelihood

$$L_N=a^N\exp\{-aS_N\}$$

Derivace

$$\frac{d}{da}L=Na^{N-1}\exp\{-aS_N\}+a^N(-S_N)\exp(-aS_N)=0$$

$$N-aS_N=0\quad\rightarrow\quad\hat{a}=\frac{N}{S_N}=\frac{1}{\bar{x}}$$

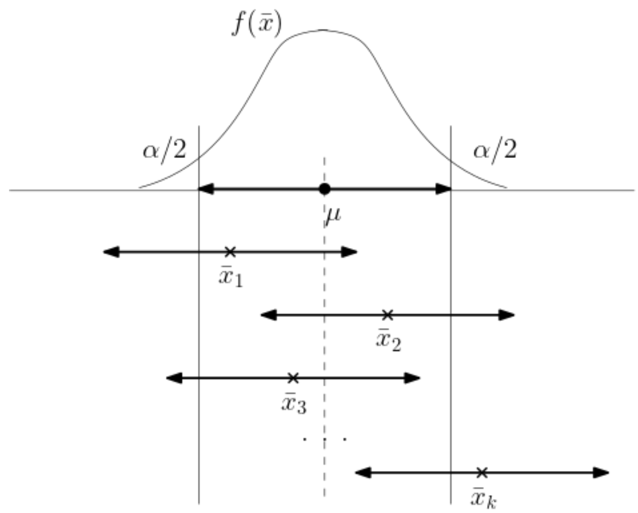
Odhady a testy se VŽDY dělají na hustotě STATISTIKY

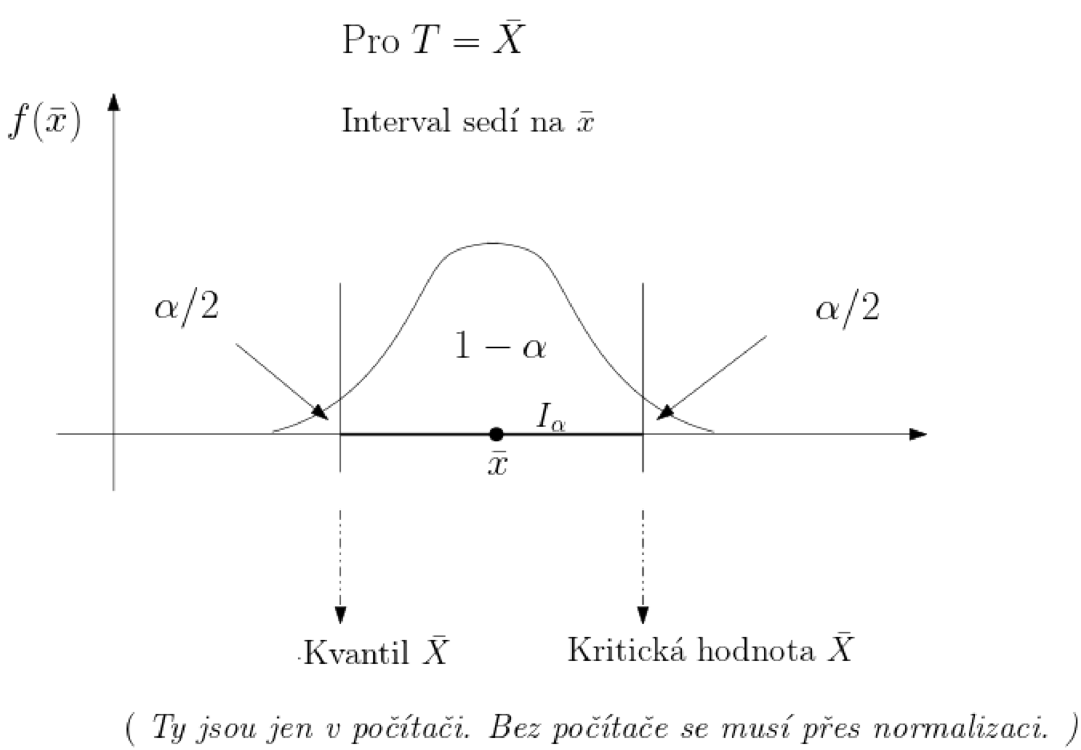
- Hodnoty náhodné veličiny X „padají“ z hustoty $f(x)$.
- Odhady parametrů „padají“ z hustoty $f(T)$, kde T je statistika pro odhad příslušného parametru.

Intervalové odhady

Intervaly spolehlivosti I_α pro parametr μ s hladinou významnosti α = interval, kde platí $P(\mu \in I_\alpha) = 1 - \alpha$

nebo je interval, kam padne $\alpha \cdot 100\%$ bodových odhadů (tj. realizací statistiky)





Bez počítače

Kvantil, kritickou hodnotu (pro normální rozdělení) známe pro normované n.v. → normujeme a normovanou veličinu označíme z

$$z = \frac{x - \mu}{\sigma}$$

pro výběrový průměr je to (N je rozsah výběru)

– normování:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{N}}} = \frac{\bar{x} - \mu}{\sigma}\sqrt{N}$$

– odnormování:

$$\bar{x} = \mu + \frac{\sigma}{\sqrt{N}} z \tag{1}$$

Hranice pro $100(1-\alpha)\%$ nejpravděpodobnějších z je

$$z \in \left(-z_{\alpha/2}, z_{\alpha/2}\right)$$

Chci $z \to \mu$ podle (1). Dolní hranice μ odpovídá dolní hranici z a to je $-z_{\alpha/2}$. Horní je $z_{\alpha/2}$. Dosadím za z a dostanu interval I_α

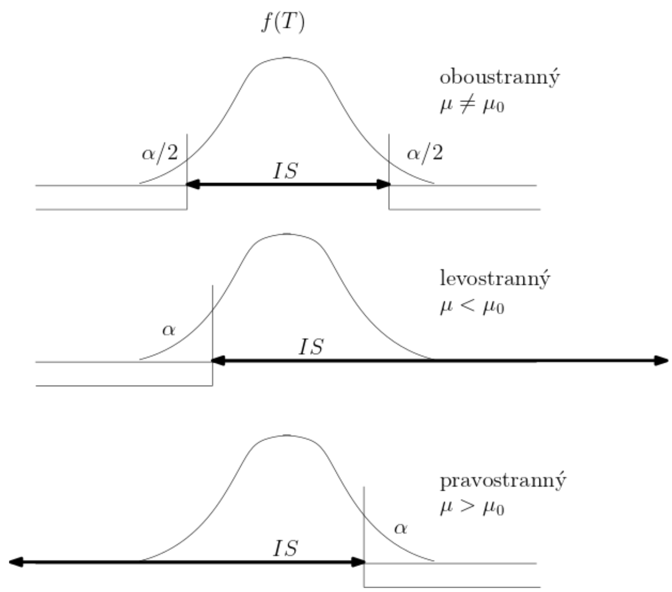
$$\mu \in \left(\bar{x} - \frac{\sigma}{\sqrt{N}} z_{\alpha/2},\, \bar{x} + \frac{\sigma}{\sqrt{N}} z_{\alpha/2}\right)$$

U jiných parametrů je to daleko složitější.

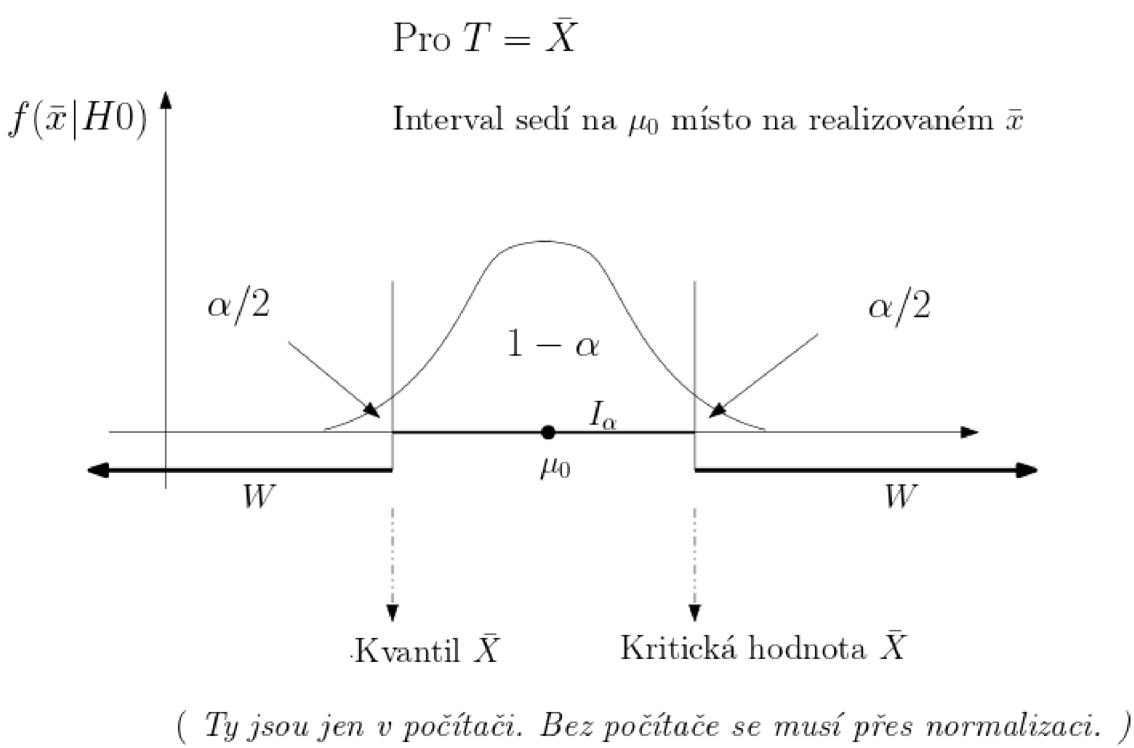
Intervaly jsou

oboustranné	$\left(\bar{x} - \frac{\sigma}{\sqrt{N}} z_{\alpha/2}, \bar{x} + \frac{\sigma}{\sqrt{N}} z_{\alpha/2}\right)$	hledám μ
levostranné	$\left(-\infty, \bar{x} + \frac{\sigma}{\sqrt{N}} z_{\alpha}\right)$	hledám množinu malých větších je jen málo
pravostranné	$\left(\bar{x} - \frac{\sigma}{\sqrt{N}} z_{\alpha}, \infty\right)$	hledám množinu velkých menších je jen málo

Směrování intervalu



Testy hypotéz

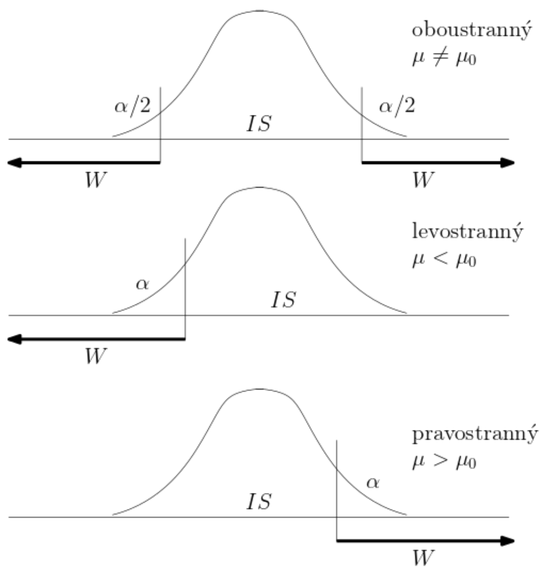


Provádějí se celé v normované oblasti (ale je to jen zvyk)

$H0$ - nulová hypotéza (ta v současnosti platí) $\mu = \mu_0$

HA - alternativní hypotéza (odporuje)	$\mu \neq \mu_0$	oboustranný
	$\mu < \mu_0$	levostranný
	$\mu > \mu_0$	pravostranný

Strany testů



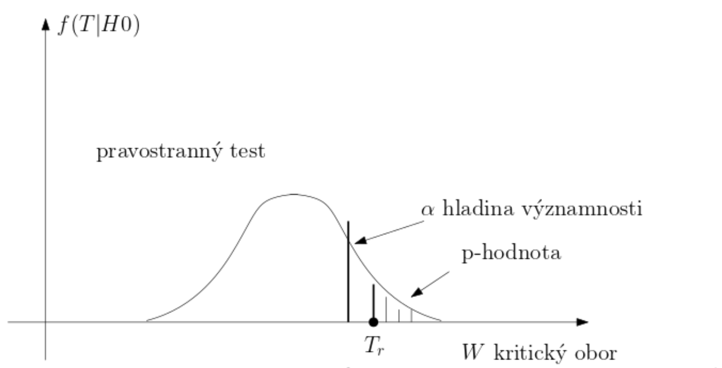
Chyby v testování

- *Chyba I druhu*: $H0$ platí a já ji zamítnu. Má hodnotu α (my jí volíme).
- *Chyba II druhu*: $H0$ neplatí a my ji nezamítneme. Značí se β a tu nemáme pod kontrolou (je dána použitým testem). Určuje tzv. *sílu testu*.

Výsledek testování (T_r je vždy počítáno pro $H0$)

- $T_r \in W$ - $H0$ zamítáme (jinak nezamítáme ale nepotvrzujeme)
  T_r realizovaná statistika, W kritický obor.
- p-hodnota malá ($pv < \alpha$) - $H0$ zamítáme
  podle velikosti pv lze určit, jak silně $H0$ zamítáme/nezamítáme.

p-hodnota pravděpodobnost, že v dalších pokusech bude hodnota statistiky stejná nebo ještě více nepříznivá jako současná.



P-hodnota pro oboustranný test

$$pv = 2\min\left\{P\left(T > T_r\right), P\left(T < T_r\right)\right\}$$

What test should I select?

Normality of the population

How many samples do I have

What is to be tested

Choice of a correct test

Testing of a parameter with normality assumption

- one sample
  - **test of expectation** testing some average value of population (either z-test if the variance is known or t-test if the variance is nor known and computed from the sample)
  - **test of variance** testing variability of data
  - **test of proportion** testing a ratio of positive results in the whole sample (e.g. ratio of drivers who exceed the admitted speed)
- two samples
  - **test of two expectations** comparing two populations (it can be either *independent* or *paired).* Tests equality of the expectations: difference eq. 0.
  - **test of two variances** Tests ratio of variances (if it is 1)
  - **test of two proportions** Equality of proportions.
- many samples
  - **ANOVA** (analysis of variance) Tests equality of expectations.

Testing of a parameter without normality assumption

Here, instead of expectation, the median (or two medians) are tested.

- one sample
  - **test of the value of the population median** Wilcoxon test
- two samples
  - **test of equality of two medians**
    - → Mann Whitney test - independent
    - → Wilcoxon test - paired
- many samples
  - Kruskal Wallis test - nonparametric ANOVA
  - Friedman test - block design

Testing of independence

Here we test independence of the two populations from which the samples have been taken.

- **Pearson test** - parametric test for continuous data
- **Spearman test** - nonparametric test for continuous data
- **Chisquare test of independence** - nonparametric test for discrete or discretized continuous data

Testing of distribution type

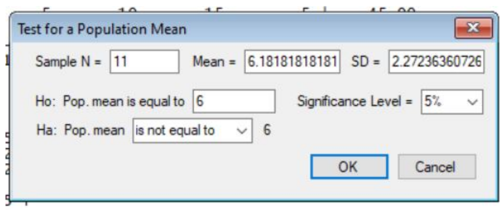
- **Normality test** - tests normality of population
- **Chisquare test of goodnes-of-fit test** - tests the type of population distribution using O (observed) and E (expected) frequencies
- **Chisquare test of homogeneity** - tests equality of two populations from frequencies of data on equal intervals

Difficult situations

Setting the evidence into Statext

The evidence (data) is mostly set into Statext by pasting (or writing) data samples into to Data window of Statext.

If instead of samples we have computed characteristics (expectation or variance) we can run the test with any data and to fill in the characteristics directly into the window opened through menu. For example: With data sample x = {5 8 9 4 9 5 7 5 9 4 3} testing the expectation we get the window like this



Here, Mean and SD are computed from data, H0 should be set as expectation according to H0. This case corresponds to testing expectation with unknown variance (variance is computed from data). In the result window we look for p-value will correspond to t distribution.

If we have the same example, but we know the variance (from technology or long observation) we can insert our standard deviation into the SD field. The resulting p-value will correspond z distribution.

If we have no data, but computed mean and standard deviation, we can set them both into the fields Mean and SD. The resulting p-value will correspond to t distribution (the variance is computed from data).

Testing two expectations as one-sided (left or right) test

Let us have two samples A and B. We want to test H0: The population with sample A has greater expectation than the other.

Here it is important, that A is the first sample and B the second one. What we test is the difference $\bar{A} - \bar{B}$ (first minus second).

Now, according to H0: $\bar{A} > \bar{B}$ from which we have $\bar{A} - \bar{B} > 0$.

The direction is set according to HA:[1] $\bar{A} - \bar{B} < 0$ which leads to left-sided test.

Chisquare tests

Chisquare tests work not with data but their frequencies of their values. If the data are discrete (categorical), we simply count how many times the individual values repeat. If they are continuous, we must choose some proper intervals of their values and count frequencies of the intervals.

So, in all cases of chisquare test we work with frequencies.

[1]HA is the opposite to H0.

Chisquare test of goodnes-of-fit needs one vector of observed frequencies (constructed from sample realization) and the second one with expected frequencies (such frequencies that exactly correspond to the tested form of distribution).

Chisquare test of homogeneity needs two vectors of frequencies. Each one from one population which are tested for equality.

Chisquare test of independence needs two vectors of frequencies. Each one from one population which are tested for independence.

Tests of hypotheses in StaText

(I) Tests with one sample

Parametric tests (normality required)

- **expectation** (known × unknown variance) - test of true average
  Ex: *A company declares that its production is more than 150 products per day. Somebody opposes and says that it is less.*
  Prg: **Parametric|Test for Population Mean ...** (solves both z-test and t-test in the Results window)
- **proportion** - test of a part from the whole
  Ex: *City manager says that only 5% of drivers exceed the permitted speed at certain street. Police is convinced that the ratio is higher.*
  Prg: **Categorical|One Sample Proportion Test ...**
- **variance** - test of variability of a variable
  Ex: *Quality of production is given by the dispersion of weight of products is. If it is higher then a given level, the machines must be adjusted. Test, if the machines are OK or it is necessary to tune them.*
  Prg: **Parmetric|Chi-Square Test for Variance ...**

Nonparametric tests (normality is not required)

- **Wilcoxon test**: tests median of rv from one sample
  – H0: median is equal to the assumed value
  – test is all sided
  Ex: *Compare caloric intake measured at 11 selected women with the recommended value 7725 kJ.*
  Prg: **Nonparmetric|One Sample|Wilcoxon Signed-Rank Test ...**

Tests of distribution type

- **test of normality**
  – H0: rv is normal
  Prg: **Parametric|Normality Tests** ... and select one
- **Kolmogorov-Smirnov test**: tests given distribution. It is based on comparison of assumed and empirical DF.
  – H0: rv has assumed distribution
  – right sided test with special crit. vals
  Prg: **Nonparametric|Two Samples (Unpaired)|Kolmogorov-Smirnov Test ...**
- **Chi-square test of homogeneity**: test of distribution type. It compares observed and expected frequencies.
  – H0: rv has the assumed distribution
  – right sided test
  Ex: *We have measured number of accidents for week days and weekends. Test if they are uniformly distributed.*
  Prg: **Categorical|Chi-Square Homogeneity Test ...**

(II) Tests with two samples

Parametric

- **two expectations** (independent × paired samples)
  Ex (indep): *Company A claims that its production is greater than that of B. Assistant of company B denies it. Test.* ... how to determine the side.
  Prg: **Parametric|Test for Two Population Means|t-Test (Independent Samples) ...**
  Ex (paired): *Uniformity of tire removal at the front wheels of cars of a specific mark has been investigated. The producer of the cars proclaims uniformity. Test it.*
  Prg: **Parametric|Test for Two Population Means|Paired t-Test**

- **two proportions**
  Ex: *Ratio of drivers violating rules in town is greater than outside. Test it.*
  Prg: **Categorical|Two Sample Proportion Test ...**

- **two variances**
  Ex: *Variability of weights of products from company A is greater than those, from company B. Test it.*
  Prg: **Parametric|Bartlett's Test for Variances ...**

Nonparametric

- **Mann-Whitney test**: tests equality of two medians (independent samples)
  – H0: the medians are equal
  – both sided test
  Ex: *Marks form math were checked at two classes of secondary school. 5 marks from the first and 8 marks from the second class were recorded. Compare the classes.*
  Prg: **Nonparametric|Two Samples (Unpaired)|Mann-Whitney U Test**

- **Wilcoxon test**: tests two medians (paired samples)
  – H0: medians are equal
  – all sided test
  Ex: *At a secondary school an improvement of students in math was checked. In the 1st class eight students were selected and their marks recorded. In the 2nd class the marks of the same students were recorded again. Test, if the results of individual students are improved.*
  Prg: **Nonparametric|Two Samples (Paired)|Wilcoxon Signed-Rank Test ...**

- **McNemar test**: tests improvement after some action. Data are yes/no - two by two table of frequencies.
  – H0: no improvement
  – right sided
  Ex: *22 selected people were tested for cold (yes/no). Then, they received some drug and after a week they were tested again. Test the effectiveness of the drug.*
  Prg: **Categorical|McNemar Test**

(III) Tests with more samples

Parametric

- **analysis of variance**: tests equality of several expectations
  – H0: expectations are equal
  – right sided test
  Ex: *Test if the power of engine of vehicles of five marks is the same.*
  Prg: **Parametric|Analysis of Variance|One-Way ANOVA ...**

- **anova with two factors**: tests equality in columns and rows.
  Ex: *Five cars are tested by three drivers. Test the cars and the drivers.*
  Prg: **Parametric|Analysis of Variance|Two-Way ANOVA (no replication) ...**

- **Bartlett test** - test of equality of more variances
  Prg: **Parametric|Bartlett's Test for Variance ...**

Nonparametric

- **Kruskal-Wallis test**: nonparametric anova.
  – H0: medians are equal
  – right sided test
  Ex: as for anova1
  Prg: **Nonparametric|Three or More Samples|Kruskal-Wallis H Test ...**

- **Friedman block test**: equality of medians
  – H0: medians are equal
  – test is right sided
  Ex. *5 shops are rated by 3 inspectors (each shop is rated by each inspector; inspectors are factors of no interest = block). Evaluate quality of the shops.*
  Prg: **Nonparametric|Block Design|Friedman Test ...**

(IV) Tests of independence

- **Test of association** of two discrete random variables.
  Ex: *We measure speed and consumption on driven cars. Is there a relation between these two variables?*
  Prg: **Categorical|Measure of Association ...**

- **Pearson test**: tests independence of two rvs. It tests correlation coefficient. (parametric test)
  – H0: rvs are independent
  – test is both sided
  Ex: *Test the data x and y if they are suitable for linear regression.*
  Prg: **Parametric|Pearson Correlation ...**

- **Spearman test**: nonparametric Pearson. Works with ranks.
  – H0: rvs are independent
  – test is both sided
  Prg: **Nonparmetric|Correlation|Spearman Rank Correlation Test ...**

- **Chi-square test of independence**: test if independence of two rvs. Compares observed and expected frequencies. Based on the definition of independence $f(x,y) = f(x)\, f(y)$.

– H0: rvs are independent
– test is right sided.
Ex: *We asked 200 people from three different areas about they pay (low, normal, high). Test if the pay depends on the area.*
Prg: **Categorical|Chi-Square Independence Test ...**

1 Interval estimates

Example 1 **(interval for μ - known variance)**

Assume, that the weight of products has normal distribution with the variance $\sigma^2 = 25$. Determine the interval α-I, $\alpha = 0.01$, in which the true weight will lie if we measured a data sample of 25 values with the average 135.

Result

$I = (132.4,\ 137.5)$ - *both sided*

Example 2 **(interval for μ - unknown variance)**

Assume, that the weight of products has normal distribution. Determine the interval α-I, $\alpha = 0.01$, in which the true weight will lie if we measured the data

x = {136 127 141 129 132 138 143 131}

Result

$I = (127.5,\ 141.8)$ - *both sided*

2 TH - one sample (parametric)

2.1 Expectation

Example 3 **(TH one expectation, unknown variance)**

On a section of motorway with a recommended speed 80 km/h we monitored the speeds of passing cars. We obtained a dataset of

a) $n = 12$ measurements

b) $n = 120$ measurements

with average $\bar{x} = 84$ and standard deviation $s = 18$. Test H0: the cars maintain the recommended speed. Test on the level $\alpha = 0.05$.

Result

influence of the sample length, BOTH-SIDED *test*

a) pv = 0.458

b) pv = 0.016

Example 4 **(TH one expectation, unknown variance)**

On a section of motorway with a recommended speed of 80 km/h we monitored the speeds of passing cars. We obtained the data

a) x = {98 86 65 92 83 92 85 66 62 82 99 92}

b) x = {98 86 65 92 83 92 85 66 62 82 99 92 89 94 81 88 79 95}

On the level $\alpha = 0.05$ test H0: the drivers in average do not exceed the speed.

Result

Test of expectation with one sample, RIGHT-SIDED

a) pv = 0.181 - they do not exceed

b) pv = 0.039 - they exceed

2.2 Proportion

Example 5 **(TH one proportion)**

On a section of motorway with a speed limit of 80 km/h we monitored the speeds of passing cars and obtained the data

x = {78 86 65 82 83 92 85 66 62 82 79 92}

Test H0 that the ratio of cars exceeding the speed is not greater than 30%

Result

n = 7, all = 12 (n - number of those who exceed)

pv = 0.016 - RIGHT-SIDED *test (normal approximation)*

$I = (0.304, 0.862)$ -> *0.3 is not in I* $\rightarrow$ *reject*

2.3 Variance

Example 6 **(TH one variance)**

A machine produces rods of a specified length. The accuracy of the machine can be verified by the variance of the lengths. If the variance is greater then the level 50, it is necessary to adjust it. A data sample of lengths has been measured

x = {101 104 103 110 108 116 129 98 104 111 115}

On the level 0.05 test if it is necessary to adjust the machine. Test on the level $\alpha = 0.05$.

Result

s2=76.2; RIGHT-SIDED *test of variance*

pv = 0.123 - not necessary

3 TH - two samples (parametric)

3.1 Expectation

Example 7 **(TH two expectations, independent)**

Two classes are to be compared in the knowledge of English. We randomly selected children from both classes and let them to write a test. The results were in the range 0 – 100 (the higher the better).

The results are

x1 = {65 81 38 76 59 58 63 63 78}

x2 = {92 83 81 96 95 42 33 66 79 85 99}

H0: "The first class is not worse than the second one". Test it on the level $\alpha = 0.05$.

Result

LEFT-SIDED TH for 2 expectations, independent samples

Comparison of groups

pv = 0.062 - H0 not rejected

Example 8 **(TH two expectations, paired)**

Teacher insists students are getting worse in English. To test if individual pupils get worse, 10 of them were selected. They wrote a test with results 0 – 100 (higher is better). Next year the same pupils wrote another, as for the level, similar test. The results are

x1 = {69 88 84 95 100 84 83 79 68 94}

x2 = {70 85 89 99 98 85 83 75 71 98}

Test on the level $\alpha = 0.05$.

Result

LEFT-SIDED TH for 2 expectations, paired samples

Comparison of individuals

pv = 0.19 - get worse

3.2 Proportion

Example 9 **(TH two proportions)**

At a crossroads, we have written down numbers of cars going straight (S) turning to left (L) and right (R). The measured data are xS=62, xL=39 a xR=46. On the level 0.05 test H0: the ratio of cars going straight is not less then those that turn.

Result

LEFT-SIDED test of 2 proportions

xT=xL+xR=85; all = 147

al = 0.0036 - H0 is rejected

4 Many samples (parametric)

4.1 Anova

Example 10 **(one-way anova)**

For five years, we monitored number of accidents at four crossroads. The results are in the following table.

$X\backslash$year	2000	2001	2002	2003	2004
X_1	2	5	3	1	2
X_2	4	2	5	6	3
X_3	2	2	5	6	4
X_4	3	3	1	4	2

a) At the level 0.05 test the hypothesis: The average number of accidents is equal at all monitored crossroads.

b) On the same level test the hypotheses: The average number of accidents is equal for both the crossroads and the years.

Result

Bartlett: pv = 0.867 and pv = 0.499 (for transposed)

a) pv_X = 0.333

b) pv_X = 0.381;   pv_year = 0.647

5 One sample (nonparametric)

5.1 Median

Example 11 (TH one median - Wilcoxon)

Let X denote the length (in centimeters), of a certain

fish species. We obtained the data set

d = {4.5 3.8 4.9 4.2 4.7 5.2 3.5}

Can we conclude that the median length of the fish species differs

significantly from the length 4.1 centimeters?

Result

Wilcoxon (one sample, BOTH-SIDED) - data are practically uniform

pv = 0.22 (0.18 - normal approximation)

6 Two samples (nonparametric)

6.1 Median

Example 12 (TH two medians - Mann-Whitney)

Two doctors recommend treating colds with two different methods. The results (number of days of the treatment) are

x1={5 6 4 4 5 8 5 7 5 6 3 4 7 7 5 6}

x2={8 4 12 9 8 3 8 15 9 6 4}

Test equality of the methods.

Result

Man-Whitney - BOTH-SIDED

H0: are equal

pv = 0.056 (are not equal)

Example 13 **(TH two medians - Wilcoxon)**

Ten athletes in some sports club were tested with respect to their performance. They all threw the javelin once and then they were subjected to an intense training. After this they threw once more. The measured lengths were

x1={68 69 75 72 83 88 79 88 76 81}

x2={71 62 81 70 74 85 82 91 85 82}

The hypothesis HA is: One day of training is not enough to improve their performance. Test on the level 0.05.

Result

Wilcoxon (two samples), RIGHT-SIDED

pv = 0.65 (0.63 or 0.64 normal approximation) - is enough (according to H0)

6.2 Detection of a change

Example 14 **(TH McNemar)**

Some cold medicine has been tested (whether it helps or harms). The health of ten selected people was inspected 0-they are OK, 1-thea have cold. Then the medicine was applied and the health checked once more. The result is

x1 = {0 0 1 0 1 1 0 1 1 1}

x2 = {0 1 1 0 1 0 0 1 0 0}

Test, if the medicine has some effect.

Result

McNemar (table in Categorical/Count nominal data)

pv = 0.62 - not reject H0: no change in samples

6.3 Chi-Square tests

Example 15 **(Chi2 independence)**

Two operators alternate regularly at two machines. The produced products are checked for quality. Each product is assigned by the machine (S) and operator (O). The following data have been measured

machine {1 1 1 2 1 2 2 2 1 2 1 2 1 2 1 1 2 2 2 1 2}

operator {2 2 1 2 1 2 2 2 1 1 2 1 1 2 2 2 1 2 2 1 2}

At the level 0.05 test the assertion that the machines and operators are with respect to the production quality independent.

Result

Chi-Square independence (table in Categorical/Count nominal data)

H0: are independent

pv = 0.28 (0.53 corrected) - are independent

Example 16 (Chi2 homogeneity)

In a study of the television viewing habits of children, a psychologist selects a random sample of 300 first graders - 100 boys and 200 girls. Each child is asked which of the following TV programs they like best: The Lone Ranger, Sesame Street, or The Simpsons. Results are shown in the contingency table below.

	Lone Ranger	Sesame Street	The Simpsons
Boys	{50	30	20}
Girls	{50	80	70}

Do the boys' preferences for these TV programs differ significantly from the girls' preferences? Use a 0.05 level of significance.

Result

(Deals with frequencies of discrete variable), H0 - are the same

pv = $6.3 \cdot 10^{-5}$ *- the preferences are not the same*

Example 17 (Chi2 goodness)

The following data are frequencies of incidents at certain big factory

time interval [hour]	8-10	10-12	12-13	13-18
number of accidents	5	8	4	12

At the level 0.05 test the hypothesis that the accidents occur uniformly.

Result

Chi-Square goodness (sum of o and e must be equal)

*i = {2 2 1 5} o = {5 8 4 12}   e = sum(o)*i/sum(i) = {5.8 5.8 2.9 14.5}*

H0: is uniform

pv = 0.62 - is uniform

7 Many samples (nonparametric)

7.1 Nonparametric anova

Example 18 (Bartlett, Kruskal-Wallis)

A factory produces some products whose weight must be constant. For the production it uses four machines. A sample of products has been taken from all machines to test equality of the product weights. The measured values are

x1={35.6 35.1 35.8 39.4 34.8}

x2={32.5 33.8 34.4 34.2 35.1 31.1}

x3={36.3 30.8 35.6 35.2 30.2 35.1 34.2}

x4={34.5 36.4 36.1 39.1 34.3 38.6}

Test the equality on condition that the data cannot be assumed normal.

Result

H0: are equal

pv = 0.033 - are not equal

further analysis - Descriptive/Box-and-Whiskers

8 Other tests

8.1 Independence

Example 19 **(Pearson)**

We would like to build a model $y = f(x)$ from the data

x = {2 5 6 8 15 21 25}

y = {12 32 41 50 115 500 650}

Test whether the data x and y are are dependent at all.

Result

Parametric/Pearson correlation

pv = 0.0017 - are dependent (H0: independent)

Example 20 **(Spearman)**

We have decided, that it is not sure, if the data prom the previous example can be considered normal. We use still a nonparametric test.

We would like to build a model $y = f(x)$ from the data

x = {2 5 6 8 15 21 25}

y = {12 32 41 50 115 500 650}

Test whether the data x and y are are dependent at all.

Result

Nonparametric/Correlation/Spearman ...

coefficient = 1 →Perfect correlation (agrees with the previous)

Example 21 **(test of normality)**

We measured the speed of cars at a given point on the road and got the data

x = {69 82 79 55 85 80 91 88 69 45 57 82 69 98}

We would like to test whether the average speed is 80 km/h but we do not know if the test should be parametric or nonparametric. Test it.

Result

H0: they are normal

pv = 0.36 - is normal

Shapiro-Wilk W test pv = 0.95; 0.64

Kolmogorov-Smirnov p > 0.1

W/S test p >0.05

Example 22 **(test of association)**

In six schools, similar classes (same year group and same number of children) were selected. Here, the number of children with excellent performance in math and English was found.

math = { 5 8 3 4 6 9}

English = {10 6 8 3 5 11}

Test whether the good performance in the subjects are associated.

Result

H0: they are associated.

chi2 - pv = 0.57 - yes, they are associated (show also in linear regression)