List of basic tasks and their brief description

Ex01simCont.sce simulation with regression model

$$y_t = b_0 u_t + a_1 y_{t-1} + b_1 u_{t-1} + e_t$$

Ex02simDisc.sce simulation with categorical model

$$f\left(y_t | u_t, y_{t-1}\right) = \begin{array}{|c||cccc|} \hline u_t & 1 & 1 & 2 & 2 \\ y_{t-1} & 1 & 2 & 1 & 2 \\ \hline \hline y_t = 1 & \Theta_{1|11} & \Theta_{1|12} & \Theta_{1|21} & \Theta_{1|22} \\ y_t = 2 & \Theta_{2|11} & \Theta_{2|12} & \Theta_{2|21} & \Theta_{2|22} \\ \hline \end{array}$$

$\cdots [u_t, y_{t-1}]$ determines column and the value of y_t corresponds to the maximal entry in this column

Ex03simStat.sce simulation with state-space model

$$x_t = M x_{t-1} + N u_{t-1} + w_t$$
$$y_t = A x_t + B u_t + v_t$$

Ex04simRegSt.sce regression model in state-space form

$$x_t = \begin{bmatrix} y_t \\ u_t \\ y_{t-1} \\ u_{t-1} \\ 1 \end{bmatrix} \quad M = \begin{bmatrix} a_1 & b_1 & a_2 & b_2 & k \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} b_0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y_t = x_t\left(1\right)$

Ex05simMix.sce simulation with mixture model

– $c \in \{1, 2, \cdots, n_c\}$ pointer

– $f_c\left(y | \cdots\right)$ components

$$f\left(c | \alpha\right) \rightarrow c = j$$
$$f_j\left(y | \cdots\right) \rightarrow y = y_t$$

Ex11EstCont.sce estimation of regression model

– $\Psi = [y_t, u_t, \cdots, 1]$

– $V = V + \Psi' \Psi$ statistics update

– $V = \begin{bmatrix} V_y & V_{yp} \\ V_{yp} & V_p \end{bmatrix} \rightarrow \hat{\theta} = V_p^{-1} V_{yp}$ parameter estimate

Ex12EstDisc.sce estimation of categorical model

– y_t, u_t, y_{t-1} data involved at time t

– code $[u_t, y_{t-1}] \rightarrow j$

– update $V\left(y_t, j\right) = V\left(y_t, j\right) + 1$

– finally normalize columns of V $\rightarrow$ parameters

Ex13EstStat.sce state estimation

– Kalman filter

[xt,Rx,yp]=Kalman(xt,yt,ut,M,N,A,B,Rw,Rv,Rx)

Ex15EstMix.sce mixture estimation

– evaluate all components $f_j\left(y_t|\cdots,\hat{\theta}_t\right)$ with substituted:

→ actual value of data y_t

→ estimates of parameters from last step

– normalize to sum equal to one

– update statistics with weighted data

$$V_c = V_c + w(c)\,\Psi'\Psi, \qquad \kappa = \kappa + w(c)$$

– compute estimates of parameters $\hat{\theta}$

Ex21preCont.sce prediction with regression model

– prior (known) are y_0, y_1

$$\hat{y}_2 = a_1 y_1 + a_2 y_0 + b u_2$$

$$\hat{y}_3 = a_1 \hat{y}_2 + a_2 y_1 + b u_3$$

$$\hat{y}_4 = a_1 \hat{y}_3 + a_2 \hat{y}_2 + b u_4$$

$$\hat{y}_5 = a_1 \hat{y}_4 + a_2 \hat{y}_3 + b u_5$$

etc.

Ex22preDisc.sce prediction with categorical model

– the model $f(y_t|u_t, y_{t-1})$ is (see above)

u_t	1	1	2	2
y_{t-1}	1	2	1	2
$y_t = 1$	$\Theta_{1\|11}$	$\Theta_{1\|12}$	$\Theta_{1\|21}$	$\Theta_{1\|22}$
$y_t = 2$	$\Theta_{2\|11}$	$\Theta_{2\|12}$	$\Theta_{2\|21}$	$\Theta_{2\|22}$

– from $[u_t, y_{t-1}]$ determine column

– $\hat{y}_t$ is index of the maximal entry of the column

Ex31ctrlCont.sce optimal control with regression model

– perform optimization (against time) → control recipes $u_t = \cdots$

– apply the control from the beginning to the end of the control interval

– for details see the program