Semester work

Example 1

1. Simulate data from the 2nd order regression model with coefficients

$$b_0 = 1,\ a_1 = 0.3,\ b_1 = 0.4,\ a_2 = 0.5,\ b_2 = 0.1,\ k = -0.1$$

and standard deviation of the noise equal to $s = 0.01$. The input is a sinusoidal signal with added normal noise w_t

$$u_t = \rho \sin(\omega t) + \vartheta w_t$$

where $\rho = 0.1$, ω is set so that the input signal performs two periods on the simulation interval which is 200 steps and $\vartheta = 0.01$. Plot the simulated data.

2. Perform estimation with a 1st order regression model of the form

$$y_t = b_0 u_t + a_1 y_{t-1} + b_1 u_{t-1} + k + e_t$$

using the simulated data. **Compare the simulated and estimated parameters**.

3. Using the simulated data and estimated model, perform zero-step prediction (i.e. at time t make prediction yp_t without using y_t).

4. Plot graph of u, y and yp and compute RPE (relative prediction error: variance$(y-yp)$/variance(y)). **Make conclusions about the estimated parameters (in comparison with the simulated ones) and about the accuracy of prediction**.

Example 2

Simulate 200 data items from discrete model $f(y_t|y_{t-1})$, $y \in \{1,2\}$ for which it holds that the probability that y changes its value is 0.1, i.e. $P(y_t = i|y_{t-1} = j) = 0.1$ for $i \neq j$.

Use the simulated data for estimation of the model with the same structure and store the point estimates of the parameters (store only the first row - why is it sufficient?).

Show the evolution of the point estimates for the following cases of the statistics initiation

1. V=1e-8*eye(2,2) - practically no prior information,
2. V =5*eye(2,2) - strong almost true information,
3. V =0.5*ones(2,2) - weak false information,
4. V=5*ones(2,2) - strong false information.

Compare the graphical results and comment them.

Example 3

For the 3rd order regression model

$$y_t = 0.8y_{t-1} + 0.2y_{t-2} + 0.1y_{t-3} + 0.1u_t + 0.4u_{t-1} + 0.1u_{t-2} - 0.2u_{t-3} + k + e_t$$

with $e_t \sim N\left(0, \sigma^2\right)$ with standard deviation $\sigma = 0.1$ perform optimal control minimizing the criterion

$$E\left[\sum_{t=1}^{N} (y_t - s_r)^2 + \omega u_t^2 + \lambda (u_t - u_{t-1})^2\right]$$

on control interval 100 steps.

Here s is a setpoint in the form

$$s_t = \text{sign}\left(10 \sin\left(18\left(1:N\right)/N\right)\right)$$

where sign$(\cdot)$ is the sign function: sign(x) is 1 for $x > 0$, is 0 for $x = 0$ and is -1 for $x < 0$.

Compare the results for various combinations of values of penalizations ω and λ. Explain the results obtained.

Hint: As a basis for your work use the program `T53ctrlX.sce`.

Example 4

Simulate 50 steps of data (x_t and y_t) from the state space model

$$x_t = 0.99x_{t-1} + u_t + w_t$$

$$y_t = x_t + v_t$$

with the initial state $x_0 = 0$ and u as a sinusoidal curve slightly corrupted by noise. The noises w_t and v_t have equal variances $1/10$.

Perform estimation of the state with the Kalman filter. For the model noise variances set the true values from the simulation. The initial state estimate variance set (i) $R_x = 1000$, (ii) $R_x = 0.0001$.

1. For both cases compare the evolution of R_x for the first several steps of the state estimation ans explain the differences.
2. Compare also the quality of state estimation and output prediction in both cases.

Hint: Consult the program: `T46statEst_KF.sce`

Example 5

We have measured data $x = [x_1, x_2, x_3]$ on a system which works in three different modes such that each mode can be described by its own model (component). Somehow, (e.g. from estimation with the teacher) we know the models of components as well as the model of the pointer which switches the working regimes. The components have normal models with expectations

$$\mu_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mu_2 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \quad \mu_3 = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

The switching model is categorical with the model probabilities

$$f_c = [0.4,\, 0.3,\, 0.3]\,.$$

Using methodology of naive Bayes, perform classification of the data and compute its accuracy.

The data are in the file: `data.csv`

Hint: Consult the program: `T65naiveB1_sim.sce`